Transferring Skills to Robots for Tasks with Cyclic Motions via Dynamical Systems Approach

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***Abstract*—The focus of this work is on robot learning of cyclic motions. The term 'cyclic' refers to motions which are repeated, but do not have a strictly defined period. The dynamics of a set of human demonstrated cyclic motions is approximated with mixtures of linear systems. The particular problems that are tackled here are: the inconsistency in periodicity of cyclic motions, occurrence of high accelerations in the transient period when reproducing the learned dynamics, and learning trajectories that involve a combination of translatory and cyclic motion components. Solutions are proposed for the aforementioned problems, and their validity is assessed through simulations. The proposed work can find implementation in learning from observation of cyclic industrial tasks (e.g., painting, peening) or service tasks (e.g., ironing, wiping).**

***Index terms*—Robot learning, dynamical systems, programming by demonstration.**

I. Introduction

TODAY'S use of robots in the industry is mainly limited to repetitive tasks in structured environments, such as applications with large series of identical parts. Among the requirements for increasing the number of robot applications is providing forms for simple, quick and flexible programming of robots for new tasks, e.g., small series of parts productions. One paradigm that provides such a framework is robot Programming by Demonstration (PbD) [1 – 4], which refers to automated robot programming from observations of tasks demonstrated by task experts.

This work concentrates on reproduction of *cyclic motions* by a robot from human demonstrations. In this context, cyclic motions refer to motions which repeat, but might not have a strictly defined period or phase [5]. Hence, the periodic motions represent a special case of cyclic motions with a constant period. This definition of cyclic motions relates to the human produced repeated movements, i.e., to the inability of humans to produce perfectly periodic movements. The motivation for learning cyclic motions stems from their frequent use for many industrial robot applications, e.g., spray painting, polishing, abrasive blasting, etc. Moreover, many potential service robot applications involve cyclic movements, e.g., ironing, wiping, cleaning.

The demonstrated cyclic trajectories are modeled here as a set of nonlinear differential equations, by using the dynamical systems approach proposed by Ijspeert *et al.* [1]. In the literature, a dynamical system with a fixed point attractor has been employed for encoding discrete movements in Ijspeert *et al.*[6], where the goal was to reach a particular state of the system. Similarly, periodic movements were encoded by dynamical systems with limit cycle behavior [7]. The demonstrated motions were modeled through two coupled dynamical systems: a canonical system which defines the phase evolution, and a transformation system which models the position, velocity and acceleration of the movements. The main advantage of using dynamical systems for robot learning consists of employing stable models which drive the system toward the attractor from different initial conditions. The learned motions are robust to perturbations and parameters variations, which is important for reproduction of learned skills in unstructured environments. Moreover, the learned motions are spatially invariant, meaning that trajectories with different amplitude, baseline and frequency can be reproduced from a learned model of the motions.

This work proposes several modifications of the dynamical systems approach in Ijspeert *et al.* [1], related to its implementation in learning industrial tasks with repeated motions. To deal with the random nature of human motions, it is proposed here to utilize the instantaneous period of the demonstrated motions in the learning step. To handle the errors in the reproduction of the motions during the transient period, a system of undamped mass-spring is employed. Lastly, most of the periodic industrial tasks involve translatory components for some of the coordinates of the motions. This case is studied here, and changing baseline of the movements is adopted to deal with the translatory components.

The paper is organized as follows. Section II provides an overview of the dynamical system approach introduced in Ijspeert *et al.* [1]. Section III discusses learning cyclic motions with a period that changes with the time evolution, which is the case for human demonstrated trajectories. Section IV proposes a dynamical system for initialization of the reproduction of cyclic motions, in order to reduce the initial errors and/or high accelerations. Section V considers learning cyclic motions with translation along some motion directions. Section VI summarizes the work.

II. Learning with Dynamical Systems

In the recent years, a body of works emerged which is oriented toward learning attractor landscapes based on the dynamics of demonstrated trajectories, as opposed to the PbD methods directed toward generating a single generalized trajectory [3]. The dynamical systems exploit the observed information for the positions, velocities and/or accelerations of the demonstrations, and approximate the nonlinear dynamic of the motion with mixtures of linear systems. The pioneering work in Ijspeert *et al.* [6] employed a set of differential equations for modeling demonstrated trajectories, which was coupled with a canonical dynamical system for controlling the phase variable of the system. Several later works [8-10] proposed certain modifications or improvements of this model. Another group of researchers working on learning the motion dynamics put the main emphasis on modeling the multivariate joint probability distribution of the demonstrated positions, velocities and/or accelerations. Statistical parametric models, such as Hidden Markov Model [11] or Gaussian Mixture Model [12-13] were used to encode the correlations between the dynamical variables of the demonstrated trajectories and to generate robot reproduction policies. The later works were focused principally on goal directed motions, and less attention was paid on the periodic motions.

The dynamical systems approach formulated in Ijspeert *et al.* [7] and its modifications [8-10] use a mass-damper-spring model:

$$M\,\ddot{x} + B\,\dot{x} + K(x_{goal} - x) = F \tag{1}$$

where x_{goal} denotes the desired steady state value of the state variable, M, B, and K are the parameters of the model, and F is a nonlinear forcing term which is learned from the demonstrated motions.

The dynamical system (1) is often written in a state space form $\dot{\mathbf{x}} = f(\mathbf{x}, \varphi, \theta)$, with states $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$, a canonical variable φ, and a set of high level parameters θ learned from the demonstrations, such as period, baseline, amplitude, etc. Thus, (1) can be rewritten as

$$\begin{aligned} \tau\dot{x}_2 &= \alpha\big(\beta\,(x_{goal} - x_1) - x_2\big) + F \\ \tau\dot{x}_1 &= x_2 \end{aligned} \tag{2}$$

where x_1 and x_2 denote the position and velocity of the trajectories, respectively, and τ is the period of the motions. The parameters α and β are chosen so that the system is critically damped (to avoid overshooting the desired values of the variables). The force term F is a nonlinear periodic function of the canonical variable φ and/or the amplitude of the motions A. The variable φ defines a canonical system which anchors the dynamical system into the phase of the oscillations, which is often formulated as a phase oscillator

$$\dot{\varphi} = \omega \tag{3}$$

where ω is the fundamental frequency component of the oscillations. Other types of oscillators can be used for the canonical system (3) as well.

The forcing term F is represented as a linear combination of a set of exponential basis functions

$$F = \left(\sum_{i=1}^{N} w_i \Psi_i \middle/ \sum_{i=1}^{N} \Psi_i \right) A \tag{4}$$

where A denotes the amplitude of the waves, and Ψ_i denote the N basis function. The following von Mises basis functions are usually adopted for learning periodic motions:

$$\Psi_i = e^{-\frac{1}{2} h \left(1 - \cos(\varphi_i - c_i)\right)} \tag{5}$$

defined by their width h and the centers c_i which are usually uniformly distributed within one period of the motion (e.g., in the range of $-\pi$ to π). The weights w_i are adjusted to fit arbitrary nonlinear functions via locally weighted linear regression in either a batch mode or an incremental mode.

The dynamical system (2) has a stable global attractor point at the baseline of the motions $\begin{bmatrix} x_1 & x_2 \end{bmatrix} \rightarrow \begin{bmatrix} x_{goal} & 0 \end{bmatrix}$, and the nonlinear periodic force F in (4) produces limit cycle oscillators.

III. Phase Warped Trajectories

To learn a model of a demonstrated task, it is assumed here that several repeated demonstrations of the same task performed under similar conditions are available. An example of a set of multiple cyclic trajectories containing the same number of waving motions is shown in Fig. 1a. The trajectories were generated by manipulating a tool with attached optical markers. An optical tracking system Optotrak Certus™ [14] was used for measuring the position and orientation of the tool with a frame rate of 0.01 seconds. It is assumed that the motions simulate a task of painting a panel with a spray gun.

When dealing with cyclic human demonstrated trajectories, the period of the motions is not constant due to the stochastic character of the human motions. It can be noticed in Fig. 1a that the period within each trajectory and between the individual trajectories is changing. On the other hand, in most of the works on dynamical systems in robot PbD, the fundamental frequency of the motion is first extracted (usually by Fourier transform approach, or even by applying a separate dynamical system for that purpose [8]), and afterwards, the first order differential equation (3) is used as a canonical system for the phase variable based on the extracted constant fundamental frequency ω. For human demonstrated cyclic motions, the constant rate of change of the phase does not correspond well to the actual phase change. One alternative in handling the different lengths and waving motions of the individual trajectories is to perform Fourier transform for each trajectory, and then to use different frequencies for the different trajectories. However, this will not compensate for the changes of the period within each trajectory. Another possibility is to scale or align the set of trajectories in order to have equal length. Among the approaches for aligning temporal data, dynamic time warping (DTW) [15] has often been used, based on reducing the norm of the squared differences between the trajectories. However, the distortion of velocities and accelerations profiles of the trajectories caused by the nonlinear warping of the time flow renders DTW approach unsuitable for trajectories analysis with the dynamical systems. Another approach which has been used for this purpose is linear time scaling of the trajectories, based on different interpolation methods [16]. The drawback of the linear time scaling approach is the sub-optimal alignment of the trajectories.

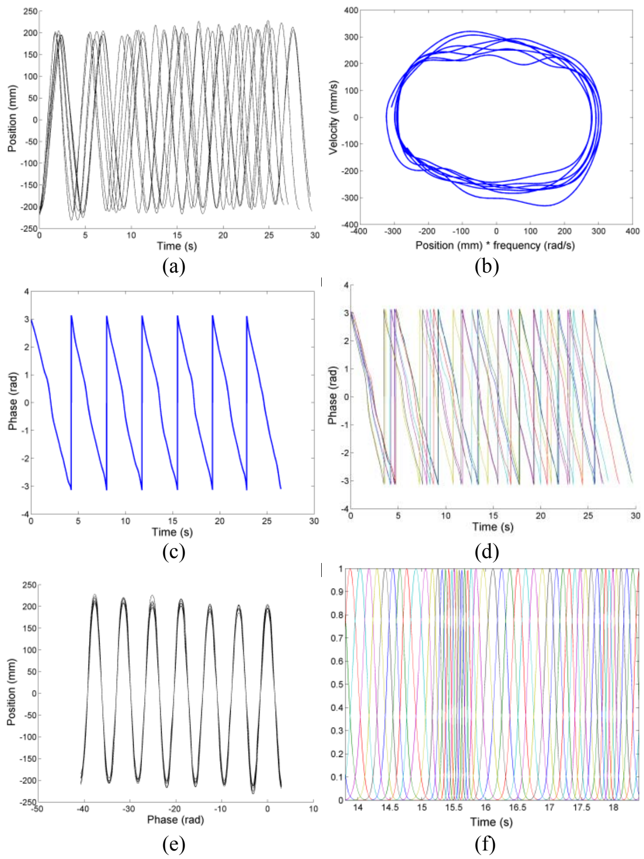


Fig. 1. (a) Non-scaled positions for the demonstrated trajectory set; (b) Phase plot for one sample trajectory; (c) Phase vs time for one of the trajectories; (d) Phase vs time for the entire set of trajectories; (e) Phase aligned trajectories; (f) Basis functions for one period of the first trajectory.

In order to take into account the changing period of cyclic motion, we propose to use learning based on the instantaneous phase of the motion. Assuming that the trajectories considered in this work can be approximated by a sinusoidal waving pattern, the phase at each time instant can be found as

$$\varphi_j^{(m)} = \tan^{-1} \frac{\dot{x}_j^{(m)}}{x_j^{(m)}\,\omega_j^{(m)}}\,, \ \text{for } m = 1, \ldots, M\,, \ \ j = 1, \ldots, T_m \quad (6)$$

where $x_j^{(m)}$ and $\dot{x}_j^{(m)}$ are the position and velocity coordinates of the m^{th} trajectory at the time moment j, $\omega_j^{(m)}$ is the instantaneous frequency of the trajectories, M denotes the total number of trajectories, and T_m is the length of the m^{th} trajectory. The instantaneous frequency $\omega_j^{(m)} = 2\pi / \tau_j^{(m)}$ is used here to scale the position coordinates, and it is found from the instantaneous time periods between the peaks across the trajectories in the demonstrated set. This parameter ensures that the phase plot of the system variables has a circular shape. For a set of 7 demonstrated trajectories shown in Fig. 1a, the plot of the scaled position versus velocity for one of the trajectories is shown in Fig. 1b. The phase evolutions for one sample trajectory and for the set of demonstrated trajectories are shown in Figs. 1cd for the phase range from π to $-\pi$. Note that by using (6), the instantaneous phase is extracted directly from the demonstrated trajectories. The dynamics of the system at each time point will be anchored in the phase variable more accurately than when using a phase oscillator system with a constant rate of change of the phase as in (3). In addition, utilizing the instantaneous phase allows cyclic motions to be learned without the need for scaling or aligning the trajectories. For comparison, the phase aligned trajectories are shown in Fig. 1e. The phase information is also important for specifying the beginning and ending portion of the trajectories, i.e., the first period and the last period of the motions. Moreover, the distribution of the basis function is not uniform in the case when using the instantaneous phase, but it is adapted to the changing phase (Fig. 1f).

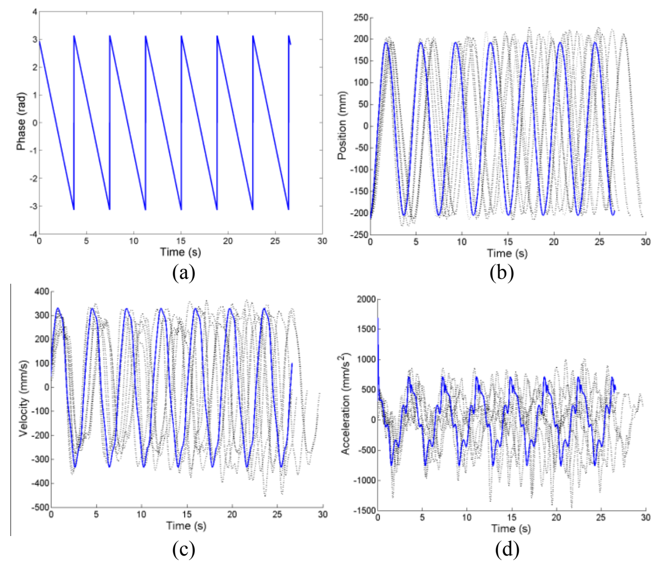


Fig. 2. (a) Phase for a constant period of the motions; (b) Positions, (c) Velocities, and (d) Accelerations for the reproduction trajectory with constant phase change. The demonstrated dynamics is shown with dashed (black) lines.

For reproduction purposes, the desired frequency of the motions can be found from the task requirements. For instance, for a painting task, the desired period of the movements can be based on the knowledge of the tool diameter and the requirement to obtain uniform distribution of the paint. In the considered task example in Fig. 1, it is assumed that a constant period of 3.773 seconds will generate the best reproduction of the demonstrated task, and the phase oscillator in (3) was used for the generating the phase. The baseline and the amplitude of the motions were extracted from the peaks and valleys of the demonstrated waves. The length of the reproduction trajectory was set equal to the length of the 4$^{\text{th}}$ demonstrated trajectory. The coefficients in (2) were set to $\alpha = 10$ and $\beta = 2.5$, the initial state was set to the means of the initial values of the demonstrated trajectories. For each trajectory, a set of 50 fixed basis functions (5) was obtained based on the instantaneous phase from (6). By concatenating the forces F and the basis functions Ψ_i from all trajectories, the weighting coefficients w_i are calculated by linear regression in a batch mode. The phase evolution is shown in Fig. 2a, while the positions, velocities and accelerations for the reproduced trajectory with the system (2-5) are shown in Figs. 2bcd.

IV. TRANSIENT PERIOD REPRODUCTION PROBLEM

One shortcoming of the approach described in Section II is the errors produced by the dynamical system at the beginning of generating a reproduction behavior. During the transient period of the described mass-damper-spring system, the dynamical system converges towards the goal state (Fig. 3), which for the position coordinate is the baseline of the oscillatory movements. During the transient period, large velocities and accelerations are produced, which might not be possible to be achieved by the available robot (e.g., initial accelerations in Fig. 2d and Fig. 3c). The speed of convergence of the system is controlled by the gains α and β in (2) (which in this work were adopted as α =10, and β = 2.5). If the transient response of the system is slowed down by adopting smaller values for the gains, then the velocities and accelerations could reach moderate values, however on the account of discrepancies between the demonstrated and reproduced motions. In this work, we propose an approach for learning cyclic motions, which does not suffer from errors in the transient period.

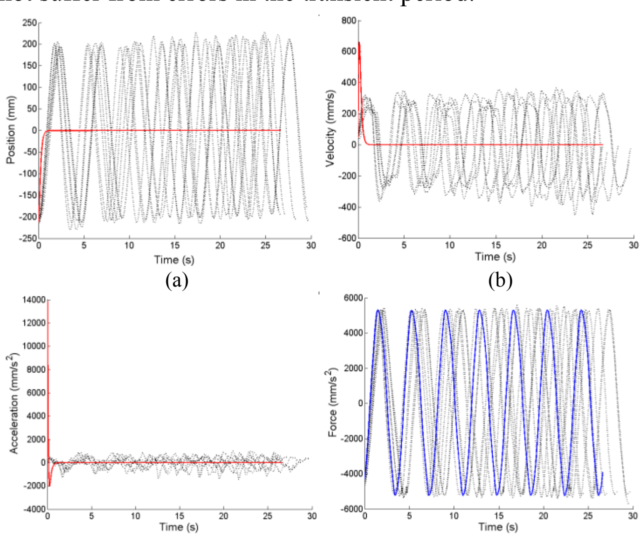


Fig. 3. The dynamics of the damped spring system without periodic force is given with the solid line for: (a) Position, (b) Velocity, and (c) Acceleration; (d) The periodic force learned by the damped spring system.

To avoid the unwanted high values of the velocities and accelerations in the starting period, we propose to use a dynamical system which consists of an undamped mass-spring system

$$\ddot{\hat{x}} = k(\hat{x}_{goal} - \hat{x}) + \hat{F}, \qquad (7)$$

during the transient period of the motions, and to switch to the dynamical system of mass-spring-damper (2) afterwards. The undamped mass-spring system is intrinsically oscillatory, and it governs the system dynamics to an oscillatory movement of the state $\hat{x}$ with the natural frequency equal to $\sqrt{k}$. For the considered system, the forcing term is found from the demonstrated motion dynamics

$$\hat{F}^{(m)} = \ddot{\hat{x}}^{(m)} - k(x_{goal} - \hat{x}^{(m)}), \quad \text{for } m = 1, \ldots, M. \qquad (8)$$

Based on the approach described in Section III, the instantaneous phase is used for learning the force function $\hat{F}_j^{(m)}$ at each time instant j, and also the frequency term k in (8) is calculated based on the period extracted from the instantaneous phase in (6).

Unlike the damped spring system in (2) which converges toward the baseline of the cyclic motions, the proposed undamped system is an oscillatory system combined with a force function which captures the local changes of the waving motions.

For the first period the centers of the Gaussian kernels were distributed between the phase at the first time step ϕ_0 and the end of the first cycle period for each trajectory m, i.e.,

$$\hat{c}_i^{(m)} \in \left\{\phi_0^{(m)}, -\pi\right\}, \text{ for } i = 1, \ldots, N, \qquad (9)$$

where N denotes the total number of basis functions used. Based on the functions $\hat{\Psi}_i^{(m)}$, and the learned forcing terms $\hat{F}^{(m)}$ in (7), the weights $\hat{w}_i$ are calculated through linear regression in a batch mode:

$$\sum_{i=1}^{N} \hat{w}_i \hat{\Psi}_i^{(m)} \Big/ \sum_{i=1}^{N} \hat{\Psi}_{i,m} = F^{(m)}, \qquad (10)$$

for $m = 1, \ldots, M$, and $i = 1, \ldots, N$.

The positions, velocities and accelerations for the first period of the motions reproduced by the undamped mass-spring system for a period of 3.773 seconds are shown with the dashed (red) line in Fig. 4abc. Compared to the trajectory of a damped system, the acceleration does not have a spike at the beginning of the time. The resulting trajectories differ, since the undamped system was trained only on the data for the first period of the motion.

It should be ensured that the transition from the undamped to the damped spring system given by (2) is smooth. However, for the damped system (2) the position coordinates have the desired values for the baseline of the motions when the phase is equal to 0 for each cycle, and the velocity reach the desired value of zero when the phase equals multiples of ±π/2, thus the system (2) will tend to generate high accelerations and velocities to reach these values. To overcome this problem here we considered the peak points of the motion as goal position coordinates x_{goal}. In this case, at phases of ±π/2, both the position and velocity will converge toward the desired values, i.e., $x \to x_{goal}$ and $\dot{x} \to 0$. Hence, the condition for the velocity equal to zero was adopted to switch from an undamaped spring system (7) to damped spring system (2), assuming that at that point the position coordinate is approximately equal to x_{goal}. The reproduction trajectory with the switching scheme is shown in Fig. 5. The transition occurs at the time equal to 1.78 seconds, when the velocity value reached zero.

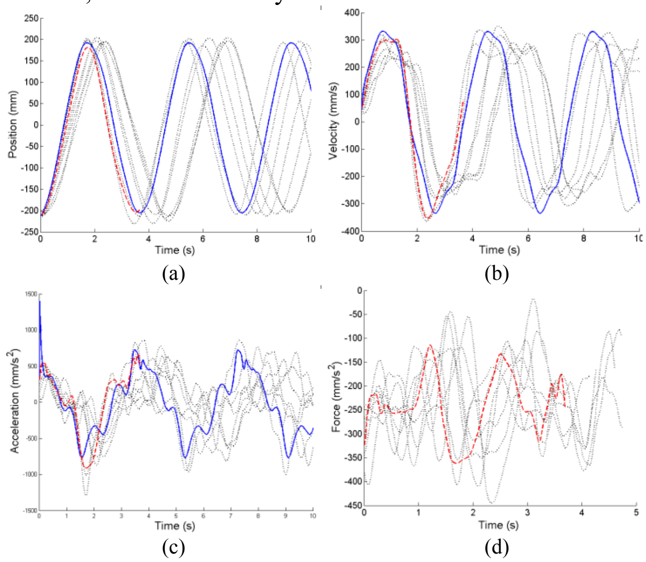


Fig. 4. (a) Position, (b) Velocity and (c) Acceleration of the dynamical system reproduced with an undamped system (dashed red line) and with a damped system (solid blue line); (d) The force learned with the undamped system for the first period.

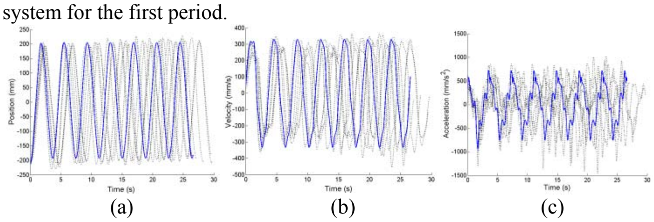


Fig. 5. Position, velocity and acceleration with a combination of an undamped spring system (7) and damped spring system (2).

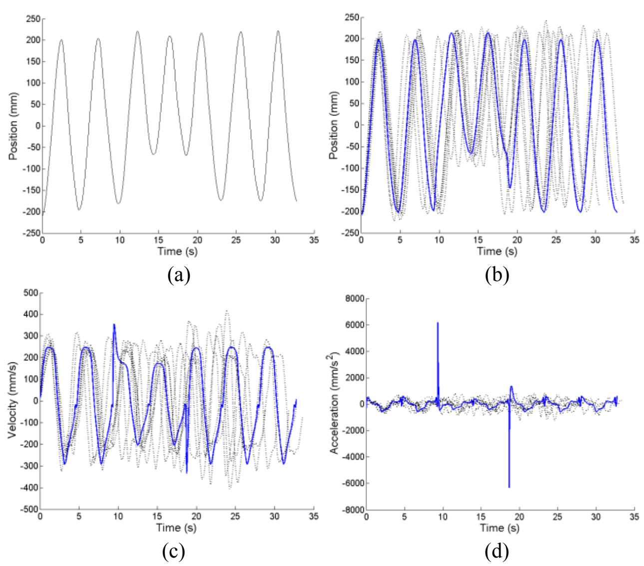


Fig. 6. (a) Position plot for one of the trajectories with changing baseline and amplitude; (b) Position, (c) Velocity, and (d) Acceleration of the dynamical system reproduced with the damped system (2).

Fig. 6a shows one sample cyclic trajectory for a different task, where the baseline and amplitude of the trajectories is modulated after the first two periods, although the frequency of the waves remains approximately the same. A damped spring system for (2) was used for reproduction with the position, velocity, and acceleration shown in Figs. 6b-d. Although the system quickly adapted to the modulated waves, the change of baseline caused high accelerations and velocities, whereas the position was mostly correctly reproduced, except for the transition at the end of the 4th period. Fig. 7 shows the reproduced trajectories with a combination of undamped and damped systems. The first half-period, as well as the half-periods of transitioning to different baseline are reproduced by an undamped system. The high accelerations produced by the changing baseline in Fig. 6d have been avoided in this case. The spikes in accelerations that occur at the beginning of each period due to the learning with the damped system (2) can be avoided by adjusting the number and width of the basis functions, as a trade-off between the accuracy and smoothness of the reproduced trajectories.

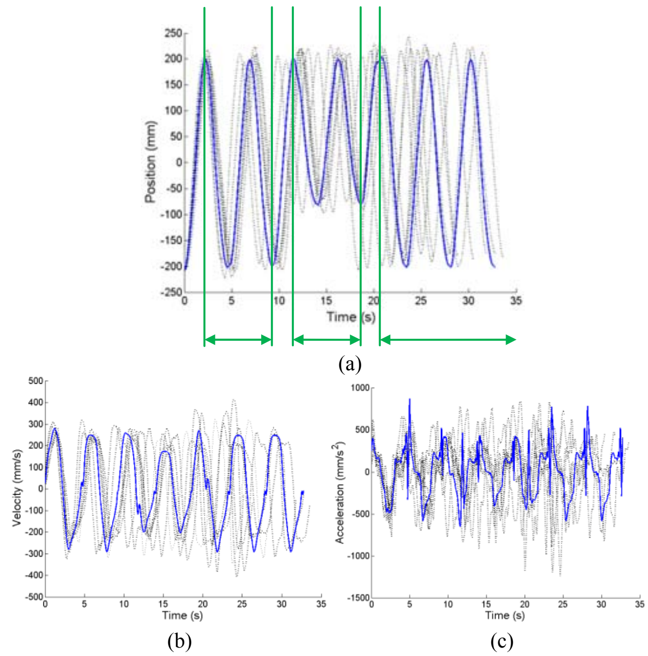
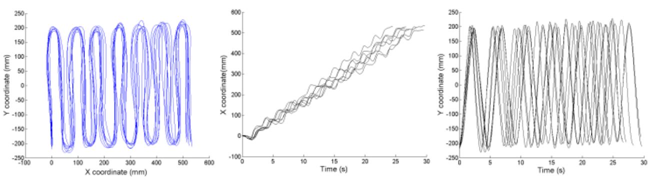


Fig. 7. Position, velocity and acceleration with a combination of an undamped spring system (7) and damped spring system (2). The parts of the trajectories between the vertical (green) lines have been reproduced with the damped spring system (2).

V. TRANSLATORY COMPONENTS OF CYCLIC MOTIONS

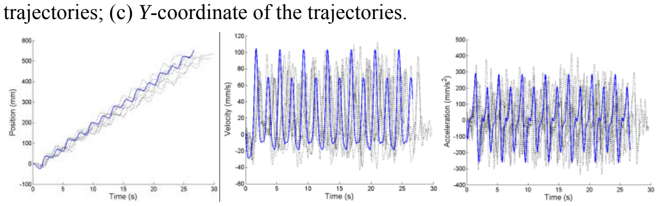
The motions considered in most of the works in the literature on learning repeated motions are closed trajectories, i.e., the trajectories ends up in the initial point at the end of each period. Here, we consider trajectories that are not closed. For instance, in the painting process the spray gun covers the part with paint but does not necessarily returns to the starting point. An example is shown in Fig. 8, where the motion of the tool in the plane of the panel is shown in Fig. 8a, whereas the horizontal (X) and the vertical (Y) coordinates in time domain are shown in Figs. 8bc. For the X-coordinate, the movement consists of a cyclic motion superimposed on a translatory motion.

In this case the baseline of the movement represents an inclined line. To learn such motions, one alternative will be to calculate the median of the motions for each period, and to use these values to specify the baseline at each period of the motion. This approach can cause high velocities and accelerations at the beginning of each period, due to the adaptation of the dynamic system to the new baseline (as it was noticed in Fig. 3). Therefore, more plausible solution is to fit a line through the peaks of the motion, in which case the baseline will be changing at each time instant. Positions, velocities and accelerations of the learned *X*-coordinate with a linear baseline by using the damped spring system (2) are shown in Fig. 9abc. As a consequence, there are discrepancies in the reproduced signal during the initiation stage, due to the adaptation step of the system (2). Fig. 10 shows the reproduced trajectories with the initial half-period reproduced by an undamped system, and switching to a damped spring system (2) afterwards. The position coordinate in Fig. 10a follows the demonstrated motions at the beginning of the reproduction. The damped system can also be used to reproduce the ending part of the motions, which may deviate from the cyclic motions as is the case in this example set (Fig. 10b).



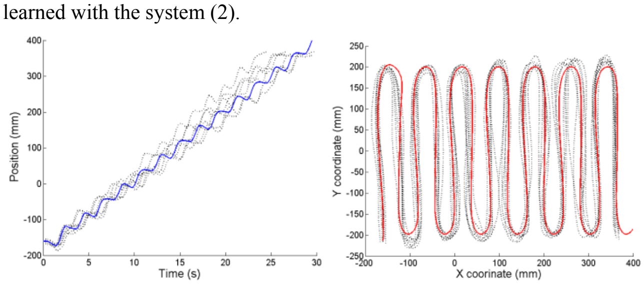
(a) (b) (c)

Fig. 8. (a) Demonstrated cyclic trajectories; (b) *X*-coordinate of the trajectories; (c) *Y*-coordinate of the trajectories.



(a) (b) (c)

Fig. 9. Position, velocity and acceleration of the *X*-coordinate from Fig. 8b, learned with the system (2).



(a) (b)

Fig. 10. (a) Position for the *X*-coordinate from Fig. 8b learned with a combination of an undamped spring system (7) and damped spring system (2); (b) Reproduced trajectory (solid red line) for the cyclic motion.

VI. Conclusion

Implementation of dynamical systems for learning cyclic motion has been studied in this work. Several modifications of the approach proposed by Ijspeert *et al.* [1] has been suggested, in order to implement this approach for transferring new skills to robots from human demonstrations. It is proposed here to extract the phase evolution directly from the demonstrations by using the instantaneous period of the motions, instead of working with the constant period which corresponds to the fundamental frequency of the motions. To reduce the high accelerations produced in the initial stage of the reproduction, an undamped spring system was proposed and implemented. Finally, cyclic motions superimposed on a translatory motion have been investigated and learned by using a linear time-changing baseline. The goal is to propose solutions for implementing the dynamical systems approaches in learning cyclic trajectories for industrial tasks.

Acknowledgment

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